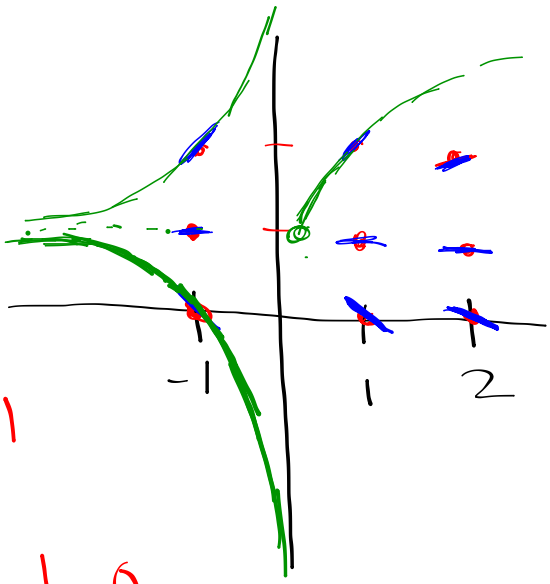
day 103

Q5) $\frac{dy}{dx} = \frac{y^{-1}}{x^2}$, $x \neq 0$

a)



$(2,2)$

$\frac{2-1}{2^2} = \frac{1}{4}$

$(-1,0)$

$\frac{dy}{dx} = \frac{0-1}{(-1)^2} = \frac{-1}{1} = -1$

$(-1,1)$ $\frac{dy}{dx} = \frac{+1-1}{(-1)^2} = \frac{0}{1}$

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day 103

q5b) $\frac{dy}{dx} = \frac{y-1}{x^2}$, $x \neq 0$

"multiply" both sides by dx
- arrange ys & xs
- No dx in denominator

separate variables

$\frac{1}{y-1}\,dy = \frac{1}{x^2}\,dx$

antidifferentiate side

$\int \frac{1}{y-1}\,dy = \int \frac{1}{x^2}\,dx$

$u = y-1$
$du = dy$

$\ln|y-1| = -\frac{1}{x} + C$

solve for C

$f(2) = 0$
$x = 2$
$y = 0$

$\ln|0-1| = \frac{-1}{2} + C$

$0 = \ln(1) = -\frac{1}{2} + C \quad \dots \quad C = +\frac{1}{2}$

$\ln|y-1| = -\frac{1}{x} + \frac{1}{2}$

solve for y = ...

$e^{\ln|y-1|} = e^{\left(-\frac{1}{x}+\frac{1}{2}\right)}$

$|y-1| = e^{-\frac{1}{x}}\left(e^{\frac{1}{2}}\right)$

watch out
Sometimes domain is important

consider $f(2) = 0 \Rightarrow$

$1-y = e^{-\frac{1}{x}}e^{\frac{1}{2}}$

$y = 1 - \left(e^{\frac{1}{2}}\right)e^{-\frac{1}{x}}$

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